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THE RECORD

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THE RECORD

American Institute of Actuaries

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FORMULA FOR CONTINUED INSTALMENTS

By LUCIUS McADAM

The extension of the method of paying a life-insurance policy by instalments, so as to provide for a continued instalment to the beneficiary in case of survival beyond the original period, has attained such a measure of approval, that the formula for the purpose is frequently demanded.

It is not available in the ordinary textbooks, but may be found quite fully elucidated in a contribution by Mr. Hugh W. Robertson to the *Transactions of the Actuarial Society of America*, Vol. VII.

As I had previously derived a formula for this purpose from another source, I thought it might be useful to the members of the American Institute of Actuaries to give it in our *Record* and show how it is derived, in as simple a manner as the case admits of. The formula comes from the book of Levi W. Meech entitled *Systems and Tables of Life Insurance*, p. 282, Example 16.

The first part of the expression there given is the Single Premium for the Instalment benefit for t years certain. The second part is the extra Single Premium which must be added to provide for the continuation of Instalment benefit to the beneficiary, in case of survival beyond the period of t years. This extra premium is derived as follows:

From the value of the life annuity to the beneficiary deferred t years, deduct the value of the chances against the beneficiary being alive at the time it would accrue.

After t years or at the end of any given year denoted by $t+n$, the payment of the instalment will depend on the combined chance that y, the beneficiary, is living and x, the insured, was alive t years previous.

This probability is represented by

$$\frac{l_{y+t+n}}{l_y} \cdot \frac{l_{x+n}}{l_x}$$

Multiplying both numerator and denominator by l_{y+t} we have

$$\frac{l_{y+t}}{l_{y+t}} \cdot \frac{l_{y+t+n}}{l_y} \cdot \frac{l_{x+n}}{l_x} = \frac{l_{y+t}}{l_y} \cdot \frac{l_{y+t+n} \cdot l_{x+n}}{l_{y+t} \cdot l_x}$$

Multiplying by the discount factor v^{t+n} and indicating the summation by Σ we have the formula

$$\Sigma v^{t+n} \cdot \frac{l_{y+t}}{l_y} \cdot \frac{l_{y+t+n} \cdot l_{x+n}}{l_{y+t} \cdot l_x}$$

Multiplying both numerator and denominator by v^y we have

$$\Sigma \frac{v^{t+n+y}}{v^y} \cdot \frac{l_{y+t}}{l_y} \cdot \frac{l_{y+t+n} \cdot l_{x+n}}{l_{y+t} \cdot l_x}$$

$$= \Sigma \frac{v^{t+y} \cdot l_{y+t}}{v^y \cdot l_y} \cdot v^n \cdot \frac{l_{y+t+n} \cdot l_{x+n}}{l_{y+t} \cdot l_x}$$

The first part may be expressed in terms of the D column, while the second part is the original formula for a joint life annuity so that the expression is equal to

$$\frac{D_{y+t}}{D_y} \cdot A_{x+t:x}$$

which must be deducted from the deferred annuity to y as stated above, hence

$$\frac{D_{y+t}}{D_y} \cdot a_{y+t} - \frac{D_{y+t}}{D_y} \cdot a_{y+t:x} = \frac{D_{y+t}}{D_y}(a_{y+t} - a_{y+t:x})$$

This is the single extra premium required. To obtain the annual premium this must be divided by the joint annuity for life, or in case of limited payments for the term of payment. That is to say, by $1 + a_{xy}$ or $1 + {}_{|n-1}a_{xy}$. We would then have the same formula on the life plan as given by Mr. Robertson, except that his formula is expressed throughout in terms of the D and N columns.

The expression $a_{y+t:x}$ in the formula strikes one at first as peculiar, since the ages x and y are never in fact t years farther apart than x and y, but the derivation of the formula explains the seeming discrepancy.

As a matter of practice it is best to compute the ordinary instalment premium in the usual manner, by multiplying the regular premium by the factor for interest. Then compute the addition necessary for the Continued Instalment, as an extra premium to be added.

These computations require the use of tables of joint life annuities.

The best ones extant for use in this country are those of Arthur Hunter based on the Makehamized American Table at 3 per cent and $3\frac{1}{2}$ per cent interest.

In using them it should be noted that the immediate annuities are always given, so that unity will drop out of the expression for the value of the annuity divisors given above. Note also that the

single life annuities must be taken from the same book where they are likewise computed from the Makehamized Mortality table. I presume it might also be contended that the entire premium should be computed from the Makehamized table. I think, however, this would be straining a point, so as to have strict mathematical accuracy at the sacrifice of convenience.

It is more convenient and sufficiently accurate to treat the extra premium to be charged for the extra benefit by itself.

It would also be more convenient to compute the reserves separately.

It appears, however, that by strict computation the reserve on the extra premium is frequently negative.

This is not a safe proposition to handle, so that probably it would simplify matters and at the same time add to safety, to compute the entire reserve as a proportion of the reserve on the original policy; that is to say, multiply the ordinary reserve upon the original policy, by a commutation factor, ascertained by dividing the gross annual premium charged by the gross annual premium for an ordinary policy of the same kind. This new factor being slightly larger than that for the ordinary instalment, provides for the practical accumulation of the extra instalment premium for reserve purposes.

NOTE ON COMMUTED VALUE OF MONTHLY INSTALMENTS CERTAIN

By E. R. CARTER

Having had occasion recently to calculate the present value of a series of monthly instalments certain at the effective rate of $3\frac{1}{2}$ per cent per annum, it occurred to me that the publication of some of the results in the *Record* might prove of value.

Present Value of 1 Due in $\frac{n}{12}$ Years $=v^{\frac{n}{12}}$					Present Value of 1 End of Each $\frac{1}{12}$ Year for $\frac{n}{12}$ Years $=a_{\overline{\frac{n}{12}}\vert}$				
$\frac{1}{12}$	.997	137	319	7460	$\frac{1}{12}$	.997	137	319	75
$\frac{2}{12}$	.994	282	834	4302	$\frac{2}{12}$	1.991	420	154	18
$\frac{3}{12}$	.991	436	520	5932	$\frac{3}{12}$	2.982	856	674	77
$\frac{4}{12}$	.988	598	354	8426	$\frac{4}{12}$	3.971	455	029	61
$\frac{5}{12}$	.985	768	313	8531	$\frac{5}{12}$	4.957	223	343	47
$\frac{6}{12}$	.982	946	374	3660	$\frac{6}{12}$	5.940	169	717	83
$\frac{7}{12}$	.980	132	513	1893	$\frac{7}{12}$	6.920	302	231	02
$\frac{8}{12}$	.977	326	707	1975	$\frac{8}{12}$	7.897	628	938	22
$\frac{9}{12}$	.974	528	933	3311	$\frac{9}{12}$	8.872	157	871	55
$\frac{10}{12}$	.971	739	168	5967	$\frac{10}{12}$	9.843	897	040	15
$\frac{11}{12}$	.968	957	390	0667	$\frac{11}{12}$	10.812	854	430	21
$\frac{12}{12}$	.966	183	574	8792	$\frac{12}{12}$	11.779	038	005	09

As an illustration of the use of these figures, let it be required to find the present value of \$100 payable at the beginning of each month for 20 years certain, at the effective rate of $3\frac{1}{2}$ per cent per annum. The present value of \$100 due at the beginning of each month for twelve months is $100[12a_{\overline{\frac{1}{12}}\vert}^{(12)}+1]$, which, from the table above, is equal to \$1181.28544. The present value of \$1 per annum due at the beginning of each year for 20 years, referring to a table of annuities certain, is found to be \$14.70983742.

$1181.28544 \times 14.70983742 = \$17{,}376.52$, which is the present value required.

FIRST-YEAR RESERVES: SOME CRITICISMS AND A SUGGESTION

By E. W. HYDE

It may seem as if the subject of first-year reserves has been pretty well threshed out; but what is the result to date? Practically chaos; the laws of the various states, which were to be harmonized on this subject, more at variance than ever. It would certainly seem that if the actuaries of the country could agree upon a system which should be generally recognized as fair and equitable, simple in application, and meeting the requirements of the life-insurance business as regards initial expenses, this system could be made legal in all the states, "a consummation most devoutly to be wished."

Let us consider in some detail the systems at present in use.

First, Level Net Premium. This system is generally acknowledged, even by the companies which use it, to be defective in that it makes no adequate provision for the necessary expenses of the first year, and, if required in all the states, it would render practically impossible the success of any new company.

Second, Full Preliminary Term. This system, while most favorable for a new company as affording the largest margin for initial expenses, has been so generally objected to, as applied to limited payment life and endowment policies, that the new laws of the various states are largely restricting its use. A company which uses it is also handicapped by the smallness of its surrender values and the derogatory statements of the agents of competing companies which use the level net premium system. In fact the words "preliminary term" have come to have attached to them in the minds of many, in my opinion unjustly, an idea of inadequacy, as if the business of a company using this system were not as well guarded and safe as that of other companies.

Third, Modified Preliminary Term. This designation is evidently indefinite, and susceptible of a great variety of meanings. In this paper, however, it will be regarded as based upon the ordinary life reserves and applying to all limited payment life and endowment policies.

As it is proposed to examine in some detail the peculiarities of these reserves, I will here simply emphasize three points.

1. The *indefiniteness* of the title, as just mentioned, which conveys no distinct meaning unless completely defined.

2. The unfortunate idea which has become associated with the name, and is, in my opinion, not at all diminished by the word "Modified."

3. The *complication* of the system. It is perfectly feasible to explain to a reasonably intelligent person what is meant by the full preliminary term reserves, and the reasons for their use, so that he will have a fair understanding of the matter, but it is very difficult for anyone but an actuary to comprehend a modified preliminary term system.

4. The so-called Select and Ultimate Reserves of M. M. Dawson. As this system appears to have found very little favor among actuaries, and has not been adopted by any state as a standard except New York, it does not require very much attention. I will merely state in passing that it appears to the writer to be even more complicated than the modified preliminary term, and that it possesses in a degree some of the same peculiarities which seem to be objectionable in that system.

Passing now to the main point, it seems quite evident that for a given kind of policy there should exist, in general, a constant or nearly constant ratio between the net premium for the first and that for subsequent years at different ages of issue, if for no other reason, in order that a uniform and sufficient percentage for agents' commission may be available at any age without exceeding the loading. Let us see how this requirement is met by the modified preliminary term system based on the ordinary life. We shall use the following notation for convenience:

$L\ N$ denotes Level Net Premium.
$M\ P$ denotes Modified Preliminary Term.
$S\ \&\ U$ denotes Select and Ultimate.
$\hat{\omega}$ = net premium first year,
π = net premium after first year,
Π = gross premium,
Λ = loading first year,
λ = loading after first year.

All values are computed by the American Table with $3\frac{1}{2}$ per cent interest.

VALUES OF $\hat{\omega}$, π, AND k FOR $M\,P$ FOR SIX POLICY FORMS

AGE	L 20			L 15			L 10		
	$\hat{\omega}$	π	k	$\hat{\omega}$	π	k	$\hat{\omega}$	π	k
21	15.05	21.53	.700	19.64	26.13	.752	29.06	35.36	.817
35	16.39	28.28	.580	22.22	34.11	.651	34.33	46.22	.740
55	22.75	51.14	.445	30.11	58.51	.515	47.48	75.88	.625

AGE	E 20			E 15			E 10		
	$\hat{\omega}$	π	k	$\hat{\omega}$	π	k	$\hat{\omega}$	π	k
21	32.93	39.40	.836	48.55	55.05	.822	80.62	87.10	.926
35	29.10	40.99	.710	44.54	56.43	.789	76.56	88.45	.865
55	26.26	54.65	.480	38.99	67.38	.579	69.05	97.44	.710

Note that in every case the ratio k is large at the younger ages (running from 70 per cent to 92.6 per cent), and diminishes as the age increases so that at age 55 we have it running from 44.5 per per cent to 71 per cent. Note also in the case of the endowment policies, while π increases with the age $\hat{\omega}$ diminishes. Let us see how this affects the first-year loading Λ_x.* The following table gives both Λ_x and λ_x as percentages of Π_x, the gross premium:

AGE	L 20		L 15		L 10		E 20		E 15		E 10	
	λ_x	Λ_x	λ_x	Λ_x	λ_x	Λ_x	λ_x	Λ_x	λ_x	Λ_x	λ_x	Λ_x
21	20.2	44.2	18	38.4	17.5	32.6	17	31.0	16	25.9	14.5	20.9
35	19.8	53.5	18	46.6	17.5	38.7	17	41.4	16	33.7	14.5	26.0
55	18.8	63.9	18	57.8	17.5	48.4	17	60.4	16	51.4	14.5	39.4

It appears that in the case of the endowment policies about twice as great a percentage of Π is available for the first-year expenses at age 55 as at age 21. Though the difference is not so great in the case of the limited payment life policies, it is still striking, and the effect of this wide variation in Λ upon the determination of a scale of first-year commissions is evident.

The following tables show the effect of the peculiarities we have noted upon the first-year reserves. A comparison is made between the $L\,N$, the $M\,P$, and the $S\,\&\,U$ systems.

* The loadings are those obtained by using the values of Π at present in use by the Columbia Life Insurance Co.

TABLES OF ${}_1V_x$ FOR SIX POLICY FORMS**

AGE	L 20			L 15			L 10		
	L N	*M P*	*S & U*	*L N*	*M P*	*S & U*	*L N*	*M P*	*S & U*
		55.3%	59%		67%	70%		79%	81%
21	14.05	7.77	8.33	18.74	12.56	13.10	28.35	22.40	22.89
35	19.58	8.09	13.10	25.49	14.16	19.13	37.73	26.81	31.63
55	32.44	5.07	18.24	39.83	12.83	26.01	57.28	31.16	44.30
		9.4%	43.6%		25%	54%		49%	
60	36.22	3.31	15.81	43.22	10.77	23.31	61.46	30.01	42.86
									57%
65							64.28		36.48

AGE	E 20			E 15			E 10		
	L N	*M P*	*S & U*	*L N*	*M P*	*S & U*	*L N*	*M P*	*S & U*
		80.8%	83.7%		87.4%	89.7%		92.7%	94.6%
21	32.71	26.43	27.37	48.91	42.73	43.85	82.14	76.18	77.67
35	32.86	21.36	26.64	48.80	37.48	42.91	81.85	70.92	76.66
		24%	61%						
55	36.13	8.76	22.10	49.20	22.19	35.82	80.02	53.89	68.13
					34%	60.3%		60%	
60				49.30	16.85	29.75	78.40	46.94	61.01
									65%
65							75.88		49.33

** The percentages given show the relation between the reserves just below them and the corresponding *L N* reserves.

This table brings out very clearly the fact that the first year *M P* reserves are for every kind of policy proportionately much too small at advanced ages of issue, and the same holds to a less degree for the *S & U*. For instance there appears to be no good reason in the case of Life 20 for setting aside the first year, at age 21 $7.77 or 29 per cent of Π and at age 55 $5.07 or 8 per cent of Π or, in the case of Endowment 20, $26.43 or 55 per cent of Π and $8.76 or 13 per cent of Π respectively at the same ages.

I desire now to suggest a mode of procedure which appears to obviate the various objectionable features to which reference has been made, and to present, so far as I can see, no new ones.

Let us asssume that for any given limited payment life, or endowment policy maturing at the end of a fixed term of years, k shall have

a fixed constant value, i.e., $\hat{\omega}_x$ shall bear a constant ratio to π_x for all values of x. Then for n payment life we shall have

$$ {}_n\pi_x = \frac{M_x}{kD_x + N_{x+1} - N_{x+n}} = \frac{A_x}{a_{x\,\overline{n}|} - (1-k)} \tag{1} $$

and $\hat{\omega}_x = k\pi_x$.

For an n payment t year endowment the formula will be

$$ {}_n\pi_{x\,\overline{t}|} = \frac{M_x - M_{x+t} + D_{x+t}}{kD_x + N_{x+1} - N_{x+n}} = \frac{A_{x\,\overline{t}|}}{a_{x\,\overline{n}|} - (1-k)} \tag{2} $$

and $\hat{\omega}_x = k\pi_x$.

Reserves computed on this plan might appropriately be called Proportional Net Premium Reserves, represented by the letters $P\,N$. If in equation (1) we make $n = 96 - x$, so that $N_{x+n} = N_{96} = 0$, we have the case of the ordinary life policy.

These formulas are about as simple and easy to use as those for the $L\,N$ system; the preliminary term notion vanishes, and the method is absolutely correct and scientific, with no guesswork as to what mortality savings for several years in advance can be safely used up in initial expenses, and no complicated dependence of one form of policy on others. The prospective reserve formula applies at once without change, while the retrospective methods must be modified to correspond with the change from $\hat{\omega}$ to π.

In order to secure uniformity in the laws of the various states as regards reserves it would only be necessary that the insurance officials and insurance companies should come to an agreement as to such values of k for the different forms of policy as would allow of a just and reasonable initial expenditure, and then secure if possible corresponding legislation. If such a *system* were generally adopted, even with values of k differing somewhat in different states, it seems to me that some relief would be afforded from the present confusion, since it would only be necessary to know the scale of k's in order to understand fully the reserve system of any state.

The following formulas give values of k which appear fair and reasonable, and do not differ greatly from the *average* values found in the $M\,P$ system.

For n payment Life Policies

$$ {}_nk = \frac{90 - 2n}{100} \tag{3} $$

For t year t payment Endowment

$$ {}_tk = \frac{90 - t}{100} \tag{4} $$

From (3) and (4) we obtain the following values of k. For Life 20, $k=0.5$; for Life 15, $k=0.6$; for Life 10, $k=0.7$; For Endt. 20, $k=0.7$; for Endt. 15, $k=0.75$; for Endt. 10, $k=0.8$.

Using these values, $\hat{\omega}$, π, and ${}_1V$ have been computed and are given below together with the corresponding LN and MP values for comparison.

Age	L 20, $k=0.5$					L 15, $k=0.6$				
	PN		LN	MP		PN		LN	MP	
	$\hat{\omega}$	π	π	$\hat{\omega}$	π	$\hat{\omega}$	π	π	$\hat{\omega}$	π
21	10.93	21.85	21.06	15.05	21.53	15.89	26.49	25.55	19.64	26.13
35	14.22	28.45	27.40	16.39	28.28	20.56	34.27	33.05	22.22	34.11
55	25.45	50.89	48.70	22.75	51.14	34.80	58.00	55.71	30.11	58.51

Age	L 10, $k=0.7$					E 20, $k=0.7$				
	PN		LN	MP		PN		LN	MP	
	$\hat{\omega}$	π	π	$\hat{\omega}$	π	$\hat{\omega}$	π	π	$\hat{\omega}$	π
21	25.25	36.07	34.77	29.06	35.56	27.87	39.81	34.94	32.93	39.40
35	32.53	46.47	44.78	34.33	46.22	28.72	41.03	40.12	29.10	40.99
55	52.60	75.14	72.26	47.48	75.88	37.45	53.50	52.21	26.26	54.65

Age	E 15, $k=0.75$					E 10, $k=0.8$				
	PN		LN	MP		MP		LN	PN	
	$\hat{\omega}$	π	π	$\hat{\omega}$	π	$\hat{\omega}$	π	π	$\hat{\omega}$	π
21	41.78	55.70	54.47	48.55	55.05	70.76	88.45	86.33	80.62	87.10
35	42.47	56.63	55.37	44.54	56.43	71.34	89.17	87.02	76.56	88.45
55	49.66	66.22	64.59	38.99	67.38	77.02	96.28	93.82	69.05	97.44

First-Year Reserves $= {}_1V$

Age	L 20			L 15			L 10		
	LN	PN	MP	LN	PN	MP	LN	PN	MP
21	14.05	3.51	7.77	18.74	8.62	12.56	28.35	18.43	22.40
35	19.58	5.78	8.09	25.49	12.45	14.16	37.73	24.85	26.81
55	32.44	7.93	5.07	39.83	17.75	12.83	57.28	36.53	31.16

FIRST-YEAR RESERVES = ${}_1V$—*Continued*

AGE	E 20			E 15			E 10		
	L N	*P N*	*M P*	*L N*	*P N*	*M P*	*L N*	*P N*	*M P*
21	32.71	21.18	26.43	48.91	35.67	42.73	82.14	65.92	76.18
35	32.86	20.96	21.36	48.80	35.36	37.48	81.85	65.50	70.92
55	36.13	21.66	8.76	49.20	33.47	22.19	80.02	58.28	53.89

Note in the last table the peculiar and obviously inappropriate values of the reserves set aside under the *M P* plan at the younger and older ages of issue, while under the *P N* plan the relative values follow quite closely the same law as under the *L N* plan.

To show this clearly the ratio of ${}_1V_{21}$ to ${}_1V_{55}$ under the different plans for several forms of policy are here given:

RATIO OF ${}_1V_{21}$ TO ${}_1V_{55}$

Reserve Plan	L 20	L 15	L 10	E 20
L N	2.31	2.12	2.02	1.10
P N	2.26	2.06	1.98	1.02
M P	0.65	1.02	1.39	0.33

The following formulas will give suitable values for k for the policies named:

$$\text{For 10 payment } t \text{ year Endt.} \ldots\ldots k = \frac{250 - t}{300} \qquad (5)$$

$$\text{For 20 payment } t \text{ year Endt.} \ldots\ldots k = \frac{160 - t}{200} \qquad (6)$$

In order to get rid entirely of the preliminary term designation we must have a suitable value of k for the ordinary life policy. This should be such that the value of $\hat{\omega}_x$ should not be greatly different from that of $A^1_{x:\overline{1}|}$, i.e., $k = \dfrac{\hat{\omega}_x}{\pi_x}$ should approximate in value to $\dfrac{A^1_{x:\overline{1}|}\, a_{x+1}}{A_{x+1}}$. The value of this last expression however varies considerably with x, being at ages 20, 40, 60, and 65 respectively 0.55, 0.39, 0.43, and 0.49. If k be taken constant and less than 0.55 we shall have a negative ${}_1V$ at some ages. It is easy to obtain a formula $k = \dfrac{4024 - 100t + t^2}{4000}$ in which $t = 96 - x$, which will give a varying

value of k equal to 0.55 at age 20, 0.39 at age 40, and 0.43 at age 60, and which will differ but slightly from the desired value at any age. I believe, however, that a constant value of k would be preferable for the sake of simplicity. If we assume: $k=0.5$, we shall have a small negative value of ${}_1V$ for each age below 25, ranging from -75 cents at 20 to nothing at 25, and small positive values at all higher ages up to 65. The four or five negative reserves are so small as to be practicably negligible, while positive ones have the values ${}_1V_{40}=2.86$, ${}_1V_{50}=5.06$, and ${}_1V_{60}=3.95$. 5.06 will be about the maximum. With the ordinary life gross premiums at p esent in use by the Columbia, and net premiums based on $k=0.5$, the first year loading would range from 62.8 per cent of II at age 20 to 60.2 per cent at age 60, averaging 61.5 per cent.

To recapitulate, the proposed system of reserves appears to present the following advantages over those at present in use:

1. Adequate and properly distributed provision for initial expenses.

2. Simplicity of formulas and processes.

3. The complete abandonment of the use of the objectionable expression "preliminary term."

4. A means by which the reserve systems of the various states might be made uniform, or, at any rate, simply and easily defined and related to each other.

In closing I may say that I shall be pleased to hear any criticisms, or answer to the best of my ability any questions bearing on the subject of this paper. I thank you sincerely for your courteous attention.

A General Formula for the Determination of $k=\frac{\hat{\omega}}{\pi}$ for any Form of Life or Endowment Policy

By E. W. HYDE

(Appendix to paper on "First Year Reserves")

In the full and modified preliminary term systems of reserves, the ratio k is a function of x, n, and t, that is, of the age at issue, the number of payments, and the term of the policy, and since in our proposed proportional system k is to be independent of x, we must assume some suitable age which is to be taken as the constant basis of the formula. A convenient value of x is $x=36$, for the reason

that, with this value, whole life policies are equivalent to 60-year endowments, and we will assume this value in our formula.

We may write in general

$$k = f(36, n, t), \tag{1}$$

and many different forms of f may be found which would answer the purpose. I have tested five or six different forms, and have finally hit upon one which is very simple, and yet covers practically the whole field of life and endowment policies. The general form of the equation is

$$k = B + C\frac{A_{36:\overline{t}|}}{a_{36:\overline{n}|}} = B + C_{n}P_{36:\overline{t}|} \tag{2}$$

in which B and C are constants to be determined by assuming values of k for two forms of policy, say ordinary life (=E. 60) and E. 10, and ${}_{n}P_{36:\overline{t}|}$ is the level net premium for the policy form under consideration for $1.00 of insurance.

If we assume $k = .46$ when $t = n = 60$, and $k = .90$ when $t = n = 10$ we find

$$k = 0.33 + 6.5\ {}_{n}P_{36:\overline{t}|} \tag{3}$$

The following table gives the corresponding values of k:

n	$t=5$	$t=10$	$t=15$	$t=20$	$t=25$	$t=30$	$t=35$	$t=40$	$t=50$	$t=60$
5						.95	.91	.88	.87	.86
10		.90	.92	.76	.71	.68	.66	.64	.63	.63
15			.69	.65	.62	.60	.58	.56	.55	.55
20				.59	.56	.54	.53	.52	.51	.51
25					.54	.52	.51	.50	.50	.50
30						.50	.50	.49	.49	.49
35							.48	.48	.48	.48
40								.47	.47	.47
50									.46	.46
60										.46

Some of the above values are obtained by interpolation. The formula will give values in excess of unity for $n = 5$, and $t = 5$, 10, 15, 20, or 25. These values of n and t are, however, very rarely required and when they are the L N reserves should be used. The formula could be easily modified so as to include these omitted cases if it seemed desirable.

GROUP VALUATION

By R. MONTAGUE WEBB

The following tables have been prepared for the purpose of facilitating the valuation of the policies of a company by the method of grouping the policies by attained age. The chief advantage of the method here outlined lies in the reduction of the number of separate groups to a minimum. The number of these groups will be precisely the number of different attained ages it is possible to make from the policy records. If, for instance, in any specified year a company had on its books insurances on persons attaining every age from 20 to 80, there would result exactly 61 groups. The method of grouping by age at issue and plan, would, under the same circumstances, result in a number of groups equal to 61 multiplied by the number of plans. As these plans would include at least whole life, 20 payment life, 15 payment life, 10 payment life, and the corresponding endowments and term policies, the number of groups would be possibly 600. The features, in this connection, of the plan for which the accompanying tables have been calculated, is the combination under one attained age of policies under all these plans, the valuation factors being applied to the sum independently of the plan.

Before deducing the basis of the method, it will be as well to describe the practical use of the tables for a check valuation. From the cards, or other records of the business in force, the total amount in force under each plan is to be entered on sheets showing the attained age at top. By previously preparing these sheets, and arranging them according to ages, the amounts and ages at issue can be filled in at once from the cards whether they be arranged for the purpose or arranged in years of issue. By referring to Table A this becomes evident. The correct net premiums and the "premium differences" can then be inserted at one operation from Tabulation No. 2 which has been arranged in parallel columns for this purpose. In using these columns it is only necessary to extract the values from the correct plan sheet at the required age. This work would be still more rapid by the preparation of similar sheets for the more usual amounts of policies, say $1,000, $2,000, $2,500 and $5,000, a very simple expedient.

Assuming then that the sheets have been so far completed, they must then be added on the machine and the three totals placed at the foot, noting that in the column of "premium differences" the amounts are subtractive, necessitating a subtotal in making the addition.

Referring now to the Valuation Sheet Tabulation No. 1, the three sets of totals are multiplied by the corresponding valuation factors shown on this sheet, the products being conveniently written below the totals. The final process is now a mere matter of adding products Nos. 1 and 3 and subtracting from this the second product. The total reserves at each attained age are now complete and may be placed directly on the adding machine.

On the assumption that the business of each year is placed on the books, on the average, on July 1, the valuation as at December 31 will be a mean valuation and the valuation as at June 30 will be a terminal valuation. The difference of valuation does not affect the insertion in the sheets of the premium and premium differences but only the selection of the valuation factors from Tabulation No. 1. It will be seen from Tabulation No. A of comparative reserves that this method of grouping and the application of the valuation factors result in an exact valuation. There must always be differences in valuations by different methods resulting from the errors in the last figures taken, both in the seriatim values and in the group valuation factors: it would, however, serve no practical purpose to carry such figures to greater length, while it would add an unnecessary amount of labor to the processes.

While paid-up policies might be included in the attained age schedules, it is better to group these separately or, where there are comparatively few of them, they may be simply listed and the reserves placed against them and added; the reserves being merely the single premiums.

The Actuaries' 4 per cent table has been used in this exhibit as a number of companies have much of their old business on this table and rate. Similar tables may be prepared on other bases, care to be taken in the preparation of the figures as to the precise nature of the valuation which the group valuation is to check, such changes being made as may be needed by reason of the valuation being full or in any way modified.

No further information is actually needed to apply these tables, and for this reason the foregoing explanation has been placed first

in this note. The basis of the method is, however, extremely simple, and of course must be understood before similar tables are compiled. Moreover, as the Institute will have an increasing number of students, it will be of use to them to explain a few points in the construction of the tables which offer ready pitfalls to the novice however enthusiastic he be, and indeed often by very reason of such enthusiasm.

The idea of this group valuation formula belongs originally to Dr. Karup and has been elaborated by Mr. Bell of the British Institute. A simplified demonstration will not be out of place.

Taking as the most general case now considered, an endowment policy with limited premiums, the value of the benefit is

$$(M_{x+1}-M_{x+n}+D_{x+n})\,/\,D_{x+1}$$

and the value of the premium, payable for m years, is

$$\pi_{x+1}(N_{x+1}-N_{x+m})\,/\,D_{x+1}$$

From the equivalence of these expressions,

$$\pi_{x+1}N_{x+1}-M_{x+1}=\pi_{x+1}N_{x+m}-M_{x+n}+D_{x+n}*$$

In these expressions the premiums are the net full preliminary term values, and π_{x+1} is the endowment premium and P_{x+1} the ordinary life premium. Coming now to the reserve at the end of t years where t is less than m, that is to say, the reserve during the premium-paying period,

$$\begin{aligned}{}_tV_x&=\{M_{x+t}-M_{x+n}+D_{x+n}-\pi_{x+1}(N_{x+t}-N_{x+m})\}/D_{x+t}\\&=A_{x+t}-\pi_{x+1}(1+a_{x+t})+(\pi_{x+1}-P_{x+1})N_{x+1}/D_{x+t}\end{aligned}$$

Here $(\pi_{x+1}-P_{x+1})$ is the difference of the premium for the endowment insurance and the whole life premium, and the factor $\dfrac{N_{x+1}}{D_{x+t}}$ may be made independent of the attained age $x+t$ by multiplying by $\dfrac{Dz}{Dz}$ where z is any convenient central age. Age 40 has been found to be a satisfactory selection. Using then D_{40}, the last term of the expression for the value becomes

$$(\pi_{x+1}-P_{x+1})\frac{N_{x+1}}{D_{40}}\cdot\frac{D_{40}}{D_{x+t}}$$

and the valuation expression is

$${}_tV_x=A_{x+t}-\pi_{x+1}(1+a_{x+t})+(\pi_{x+1}-P_{x+1})\frac{N_{x+1}}{D_{40}}\cdot\frac{D_{40}}{D_{x+t}}$$

*Also $M_{x+1}=P_{x+1}N_{x+1}$ and $\pi N_{x+1}-M_{x+1}=(\pi_{x+1}-P_{x+1})N_{x+1}$.

Considering this expression for a moment, it is at once seen that there are three terms each composed of two factors: in each term one factor depends upon the age at issue and the other factor is dependent upon the attained age. Thus the amount insured has the factor A_{x+t}, the net premium has the factor $(1+a_{x+t})$, and the difference of premiums has the factor $\frac{D_{40}}{D_{x+t}}$. This is the form of expression sought, and by a similar process it will be found that the same expression for the reserve applies to limited payment life policies, to ordinary endowment insurances, and to term policies, using the corresponding net premiums, and noticing that in the case of term policies the last term becomes negative.

The tabulations herewith submitted contain the net premiums for the ten cases considered, with the differences already modified by the factor for age at issue, arranged in convenient proximity for use. The valuation factors, or the attained age factors are also tabulated and arranged in order of the terms of the valuation expression. In the case of a mean or midyear valuation, the sum insured, premium, and modified premium difference remain the same as in the case of a terminal reserve; the three valuation factors are, however, different. These also have been calculated for this note and are given in correct order for application in Tabulation No. 1. In obtaining the correct expression for the mean reserve, the mean values are required of the single premium, the annuity due, and the factor of the premium difference. The student should notice in this connection that while the mean single premium is one-half the sum of the two adjacent annual values, this is not the case with the annuity due: that is to say, the mean *required* between $(1+a_y)$ and $(1+a_{y+1})$ is not half their sum, but $\frac{1}{2}+\frac{1}{2}(a_y+a_{y+1})$.

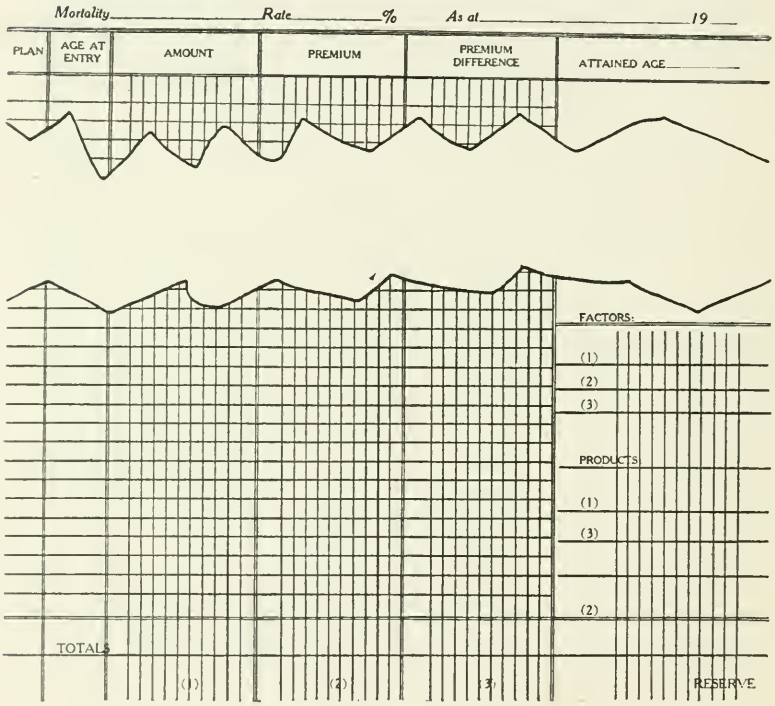
The mean value expression will then be found to be

$$ {}_{t+\frac{1}{2}}V_x \equiv A_{x+t+\frac{1}{2}} - \pi_{x+1}(\tfrac{1}{2}+A_{x+t+\frac{1}{2}}) + \text{Prem. Diff.} \times \tfrac{1}{2} \left\{ \frac{D_{40}}{D_{x+t}} + \frac{D_{40}}{D_{x+t+1}} \right\} $$

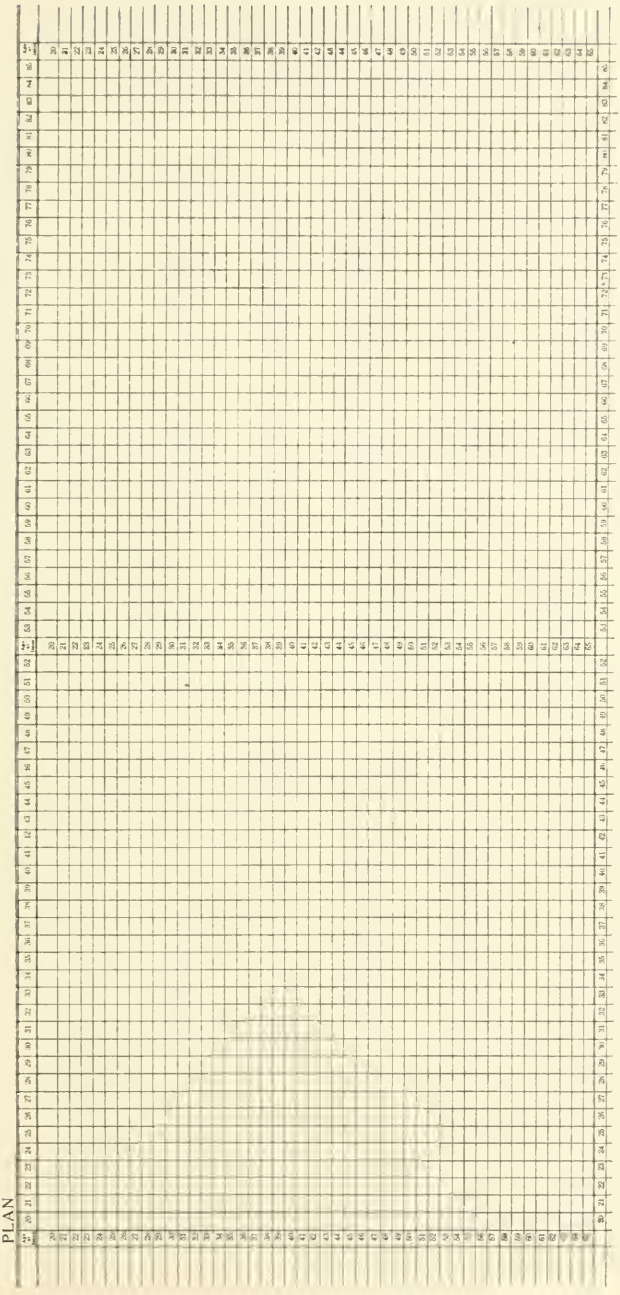
In conclusion it is worth while to draw attention to the fact that in searching for a method of valuing the policies of a company by grouping them, we seek a method which is both rapid and easy of application, a method which is shorter and less laborious than the seriatim valuation which the group method is intended to check.

With this in mind it is well to examine the methods that from time to time suggest themselves and apply this test to them, as many theoretically excellent methods of valuation entirely fail to fulfil the two essential requirements. For the same reasons all reference has been purposely omitted to the application of the formula of this note to insurances containing varying premiums or varying insur-

"GROUP VALUATION BY ATTAINED AGE.



ance, such as increasing premium insurances and insurances with return of premium. Experience teaches that the use of complicated expressions is neither rapid nor easy and moreover leads to the entry of errors which are difficult to detect and tedious to rectify. The extension of this method to such varying insurances has been taken up by Mr. Robert Henderson in No. 42 of the *Transactions of the Actuarial Society* and other methods of grouping have been deduced by Mr. Whiting and others from time to time.


PLAN

A

COMPARISON OF RESERVES

ATTAINED AGE 45—PRELIMINARY TERM VALUATION

(Actuaries' 4%)

PLAN	AGE AT ENTRY	AMOUNT	PREMIUM	PREM. DIFF.
Ord. Life.....	29	7,500	127.28	0.00
20 Pay. Life	30	5,000	123.30	992.05
15 Pay. Life	35	2,000	67.36	541.98
10 Pay. Life	38	1,000	50.13	468.19
20 Year End.....	28	5,000	200.80	3,670.35
15 Year End.....	34	4,000	232.44	3,349.80
10 Year End	36	2,500	239.70	3,619.75
20 Year Term ...	33	3,000	32.01	−587.85
15 Year Term ...	39	2,000	25.44	−352.76
10 Year Term ...	40	2,500	29.60	−481.10
5 Year Term ...	43	5,000	60.30	−966.50
Totals.....		$39,500	$1,188.36	$10,253.91

39,500 × .42857	=$16,928.52	39,500 × .43371	=$17,131.55
10,253.91×1.2856	= 13,182.44	10,253.91×1.31957	= 13,530.75
	$30,110.96		$30,662.30
1188.36 ×14.8565	= 17,654.87	88.36×14.222	= 16,900.86
Terminal Reserve.....	$12,456.09	Mean Reserve.....	$13,761.44

SERIATIM VALUATION

Ord. Life.....	29	7,500	1,323.15	1,442.40
20 Pay. Life	30	5,000	1,585.90	1,723 45
15 Pay. Life	35	2,000	552.88	624.28
10 Pay. Life	38	1,000	285.63	338.45
20 Year End	28	5,000	3,879.10	4,156.80
15 Year End	34	4,000	2,568.16	2,849.92
10 Year End	36	2,500	2,164.15	2,451.92
20 Year Term ...	33	3,000	54.42	70.14
15 Year Term ...	39	2,000	25.68	40.08
10 Year Term ...	40	2,500	13.12	28.42
5 Year Term ...	43	5,000	4.00	35.55
Totals.....		$39,500	$12,456.19	$13,761.41

GROUP VALUATION BY ATTAINED AGES

Tabulation No. 1—Valuation Factors

(Actuaries' 4% Preliminary Term)

Attained Age	Terminal			Mean			Attained Age
	Amount	Premium	Premium Difference	Amount	Premium	Premium Difference	
20	.25191	19.450	.38487	.25423	18.889	.39403	20
21	.25656	19.329	.40320	.25897	18.766	.41282	21
22	.26138	19.204	.42245	.26387	18.639	.43255	22
23	.26636	19.075	.44265	.26893	18.508	.45326	23
24	.27150	18.941	.46387	.27416	18.372	.47501	24
25	.27682	18.803	.48615	.27957	18.231	.49785	25
26	.28231	18.660	.50956	.28515	18.086	.52185	26
27	.28799	18.512	.53415	.29093	17.936	.54707	27
28	.29386	18.360	.56000	.29688	17.781	.57358	28
29	.29991	18.202	.58716	.30304	17.621	.60145	29
30	.30617	18.040	.61575	.30939	17.456	.63078	30
31	.31262	17.872	.64582	.31595	17.285	.66164	31
32	.31929	17.698	.67746	.32273	17.109	.69412	32
33	.32617	17.520	.71078	.32972	16.927	.72832	33
34	.33327	17.335	.74586	.33693	16.739	.76433	34
35	.34060	17.144	.78281	.34438	16.546	.80228	35
36	.34817	16.948	.82176	.35208	16.346	.84228	36
37	.35599	16.744	.86281	.36003	16.139	.88445	37
38	.36407	16.534	.90610	.36824	15.925	.92895	38
39	.37241	16.317	.95180	.37072	15.705	.97590	39
40	.38104	16.093	1.00000	.38550	15.477	1.02544	40
41	.38996	15.861	1.05089	.39457	15.241	1.07777	41
42	.39918	15.621	1.10465	.40394	14.997	1.13307	42
43	.40871	15.374	1.16149	.41361	14.746	1.19159	43
44	.41852	15.119	1.22169	.42354	14.488	1.25364	44
45	.42857	14.856	1.28560	.43371	14.223	1.31957	45
46	.43886	14.590	1.35355	.44410	13.953	1.38977	46
47	.44935	14.317	1.42600	.45468	13.678	1.46468	47
48	.46002	14.039	1.50336	.46545	13.398	1.54973	48
49	.47088	13.757	1.58611	.47639	13.113	1.63044	49
50	.48191	13.470	1.67478	.48751	12.824	1.72238	50
51	.49311	13.179	1.76998	.49928	12.531	1.82120	51
52	.50446	12.884	1.87242	.51020	12.234	1.92766	52
53	.51595	12.585	1.98291	.52176	11.934	2.04263	53
54	.52757	12.283	2.10236	.53344	11.630	2.16707	54
55	.53931	11.978	2.23197	.54523	11.324	2.30212	55
56	.55116	11.670	2.37246	.55713	11.014	2.44911	56
57	.56310	11.359	2.52577	.56912	10.702	2.60952	57
58	.57514	11.046	2.69327	.58120	10.388	2.78509	58
59	.58726	10.731	2.87691	.59334	10.073	2.97793	59
60	.59943	10.415	3.07895	.60553	9.756	3.19062	60
61	.61163	10.098	3.30229	.61773	9.439	3.42622	61
62	.62382	9.781	3.55016	.62991	9.123	3.68836	62
63	.63600	9.464	3.82656	.64206	8.806	3.98134	63
64	.64812	9.149	4.13613	.65414	8.492	4.31039	64
65	.66017	8.836	4.48466	.66615	8.180	4.68189	65
66	.67212	8.525	4.87913	.67803	7.871	5.10355	66
67	.68395	8.217	5.32798	.68980	7.565	5.58489	67

TABULATION NO. 1—VALUATION FACTORS—*Continued*

Attained Age	Terminal			Mean			Attained Age
	Amount	Premium	Premium Difference	Amount	Premium	Premium Difference	
68	.69565	7.913	5.84180	.70142	7.263	6.13758	68
69	.70719	7.613	6.43336	.71288	6.965	6.77589	69
70	.71857	7.317	7.11842	.72416	6.672	7.51783	70
71	.72976	7.026	7.91725	.73526	6.383	8.38622	71
72	.74077	6.740	8.85520	.74617	6.099	9.41000	72
73	.75157	6.459	9.96480	.75686	5.821	10.12620	73
74	.76215	6.184	11.28761	.76733	5.549	12.08306	74
75	.77251	5.915	12.87851	.77757	5.283	13.84363	75
76	.78264	5.651	14.80876	.78759	5.045	15.99088	76
77	.79254	5.394	17.17300	.79737	4.768	16.63676	77
78	.80220	5.143	20.10052	.80739	4.521	21.93381	78
79	.81159	4.899	23.76711	.81616	4.280	26.09027	79
80	.82074	4.661	28.41343	.82519	4.045	31.39502	80
81	.82965	4.429	34.37661	.83400	3.816	38.25432	81
82	.83836	4.203	42.13204	.84263	3.591	47.24730	82
83	.84691	3.980	52.36257	.85112	3.371	59.22208	83
84	.85534	3.761	66.08160	.85952	3.152	75.44725	84
85	.86370	3.544	84.81290	.87685	2.936	97.88750	85
86	.87200	3.328	110.96210	.87612	2.721	129.66157	86
87	.88024	3.114	148.42105	.88433	2.507	176.06107	87
88	.88842	2.901	203.70109	.89246	2.296	246.01654	88
89	.89650	2.691	288.33200	.90045	2.088	356.05450	89
90	.90441	2.485	423.77700	.90827	1.884	537.74070	90

TABULATION NO. 2—PREMIUMS AND PREMIUM DIFFERENCES
(Actuaries' 4% Preliminary Term)
Life

AGE AT ENTRY	20 PAYMENT LIFE		15 PAYMENT LIFE		10 PAYMENT LIFE	
	Premium	Premium Difference	Premium	Premium Difference	Premium	Premium Difference
20	19.98	321.67	24.44	535.48	34.13	1,000.08
21	20.37	307.31	24.91	513.69	34.79	962.82
22	20.78	293.89	25.40	492.98	35.46	926.48
23	21.21	280.93	25.91	472.85	36.16	891.38
24	21.65	268.03	26.44	453.29	36.89	857.48
25	22.11	255.61	26.99	434.31	37.64	823.40
26	22.58	243.30	27.56	415.89	38.42	792.29
27	23.07	231.80	28.15	398.35	39.22	761.28
28	23.58	220.10	28.75	380.37	40.05	730.67
29	24.11	209.18	29.38	363.58	40.91	716.05
30	24.66	198.41	30.04	347.29	41.80	672.72
31	25.23	187.83	30.71	330.99	42.72	644.73
32	25.83	177.71	31.41	315.25	43.67	617.44
33	26.47	168.50	32.14	300.28	44.66	591.26
34	27.10	158.34	32.89	285.14	45.68	565.25
35	27.77	149.11	33.68	270.99	46.73	540.12
36	28.48	140.12	34.50	256.94	47.82	515.43
37	29.22	131.38	35.35	243.24	48.95	491.41
38	30.00	123.09	36.24	230.07	50.13	468.19
39	30.82	114.90	37.17	217.10	51.36	445.46
40	31.69	107.16	38.15	204.66	52.63	423.21
41	32.60	99.70	39.18	192.75	53.97	401.90
42	33.57	92.52	40.27	181.20	55.37	381.10
43	34.60	85.64	41.40	169.79	56.83	360.74
44	35.67	78.93	42.59	158.90	58.34	340.92
45	36.81	70.89	43.84	148.32	59.91	321.53
46	38.00	66.46	45.13	138.05	61.53	302.71
47	39.25	60.52	46.48	128.04	63.20	284.18
48	40.56	54.92	47.88	118.45	64.93	266.40
49	41.95	49.71	49.35	109.22	66.72	248.93
50	43.41	43.66	50.88	100.29	68.56	231.94
51	44.95	39.82	52.48	91.72	70.47	215.51
52	46.58	35.41	54.15	83.46	72.44	199.55
53	48.31	31.32	55.91	75.72	74.48	184.22
54	50.14	27.48	57.76	68.38	76.59	169.43
55	52.09	23.91	59.71	61.39	78.79	155.24
56	54.17	20.69	61.76	54.82	81.06	141.62
57	56.38	17.68	63.93	48.64	83.44	128.66
58	58.75	15.03	66.23	42.93	85.93	116.42
59	61.29	12.59	68.68	37.61	88.53	104.76
60	64.00	10.49	71.28	32.75	91.25	93.81
61	66.91	8.62	74.04	28.27	94.09	83.50
62	70.02	6.97	76.98	24.19	97.07	73.88
63	73.33	5.51	80.11	20.50	100.21	64.97
64	76.94	4.37	83.45	17.20	103.50	56.70
65	80.78	3.37	87.10	14.41	106.96	49.11

TABULATION NO. 3—PREMIUMS AND PREMIUM DIFFERENCES
(Actuaries' 4% Preliminary Term)
Endowment

AGE AT ENTRY	20 YEAR ENDOWMENT		15 YEAR ENDOWMENT		10 YEAR ENDOWMENT	
	Premium	Premium Difference	Premium	Premium Difference	Premium	Premium Difference
20	39.41	1,253.10	56.79	2,086.31	94.57	3,897.44
21	39.49	1,176.47	56.85	1,965.65	94.62	3,682.63
22	39.57	1,103.57	56.91	1,850.80	94.68	3,478.38
23	39.66	1,034.29	56.99	1,741.94	94.74	3,283.38
24	39.74	967.68	57.07	1,637.97	94.80	3,197.25
25	39.84	904.88	57.14	1,538.41	94.86	2,919.71
26	39.94	844.95	57.23	1,444.99	94.93	2,752.31
27	40.04	788.16	57.31	1,354.35	95.01	2,590.34
28	40.16	734.08	57.42	1,269.10	95.09	2,436.83
29	40.28	682.90	57.51	1,187.66	95.17	2,290.95
30	40.42	634.53	57.62	1,110.48	95.26	2,152.05
31	40.56	588.31	57.73	1,036.86	95.35	2,019.65
32	40.72	544.72	57.85	966.94	95.44	1,893.46
33	40.90	503.86	57.97	900.59	95.54	1,773.75
34	41.09	464.72	58.11	837.46	95.65	1,659.58
35	41.30	428.12	58.27	778.11	95.76	1,551.26
36	41.54	393.55	58.44	721.52	95.87	1,447.88
37	41.80	360.93	58.63	668.02	96.00	1,349.91
38	42.09	330.36	58.85	617.66	96.15	1,257.10
39	42.42	301.58	59.10	570.01	96.32	1,169.00
40	42.80	274.84	59.38	525.09	96.51	1,085.49
41	43.21	249.74	59.69	482.77	96.74	1,006.70
42	43.68	226.33	60.06	443.14	97.01	932.21
43	44.20	204.44	60.47	405.78	97.32	861.79
44	44.77	184.09	60.92	370.72	97.67	795.44
45	45.41	165.24	61.43	337.92	98.05	732.63
46	46.10	147.79	61.97	307.12	98.47	673.57
47	46.86	131.59	62.57	278.30	98.93	617.85
48	47.69	116.80	63.22	251.43	99.43	565.77
49	48.59	103.11	63.94	226.57	99.98	516.44
50	49.57	90.54	64.72	203.35	100.58	470.36
51	50.65	79.13	65.57	181.80	101.23	427.17
52	51.82	68.67	66.50	161.85	101.94	386.78
53	53.10	59.30	67.52	143.56	102.71	349.16
54	54.52	51.09	68.64	126.77	103.56	314.18
55	56.06	43.43	69.87	111.37	104.48	281.61
56	57.74	36.74	71.21	97.32	105.50	251.54
57	59.58	30.80	72.69	84.56	106.62	223.73
58	61.59	25.63	74.32	73.11	107.86	198.22
59	63.79	21.14	76.11	62.75	109.22	174.74
60	66.18	17.15	78.07	53.52	110.72	153.35
61	68.80	13.83	80.22	45.29	112.37	133.86
62	71.64	10.98	82.57	38.01	114.17	116.17
63	74.31	7.67	85.14	31.63	116.15	100.22
64	78.08	6.62	87.94	26.00	118.31	85.88
65	81.72	5.01	91.00	21.23	120.67	73.05

TABULATION NO. 4—PREMIUMS AND PREMIUM DIFFERENCES
(Actuaries' 4% Preliminary Term)
Term

Age at Entry	20 Year		15 Year		10 Year		5 Year	
	Premium	Premium Difference	Premium	Premium Difference	Premium	Premium Difference	Premium	Premium Difference
20	8.02	251.68	7.73	265.58	7.47	278.05	7.25	288.59
21	8.16	247.75	7.86	261.39	7.58	274.12	7.34	285.03
22	8.30	243.90	7.99	257.26	7.69	270.19	7.45	280.53
23	8.45	240.10	8.14	252.75	7.78	267.46	7.51	278.48
24	8.64	235.15	8.28	249.08	7.96	261.46	7.65	273.45
25	8.81	231.44	8.43	245.35	8.09	257.80	7.77	269.52
26	9.01	227.13	8.60	241.35	8.23	254.18	7.89	265.97
27	9.23	221.95	8.77	237.04	8.39	249.49	8.03	261.30
28	9.44	218.23	8.95	233.42	8.55	245.82	8.17	257.60
29	9.73	212.10	9.15	229.09	8.73	241.40	8.34	252.82
30	10.01	206.99	9.36	224.97	8.93	236.87	8.49	249.05
31	10.31	201.94	9.59	220.75	9.10	233.55	8.66	245.04
32	10.67	195.95	9.84	216.41	9.29	229.96	8.84	241.06
33	11.05	189.88	10.13	211.26	9.49	226.13	9.04	236.59
34	11.48	183.74	10.46	206.08	9.72	222.28	9.21	233.45
35	11.94	177.36	10.81	200.66	9.97	217.98	9.41	229.53
36	12.45	170.97	11.21	195.03	10.25	213.66	9.64	225.50
37	13.02	164.22	11.65	189.22	10.56	209.11	9.85	222.06
38	13.63	157.54	12.15	182.92	10.93	203.83	10.08	218.40
39	14.32	150.63	12.72	176.38	11.35	198.43	10.36	214.36
40	15.06	143.84	13.32	170.10	11.84	192.44	10.68	209.94
41	15.90	136.46	14.00	163.33	12.37	186.38	11.05	205.04
42	16.81	129.32	14.76	156.45	12.99	179.88	11.51	199.47
43	17.81	122.14	15.59	149.61	13.66	173.50	12.06	193.30
44	18.86	115.33	16.53	142.25	14.43	166.52	12.62	187.44
45	20.10	107.57	17.53	135.28	15.26	159.74	13.34	180.44
46	21.40	100.20	18.61	128.21	16.17	152.71	14.10	173.49
47	22.79	93.20	19.78	121.31	17.19	145.50	14.92	166.70
48	24.32	85.95	21.09	113.96	18.33	137.90	15.77	160.10
49	25.91	79.30	22.48	106.89	19.44	131.34	16.74	156.27
50	27.68	72.45	24.00	99.85	20.65	124.79	17.80	146.02
51	29.58	65.85	25.65	92.89	22.03	117.80	19.10	137.96
52	31.61	59.60	27.44	86.07	23.47	111.26	20.17	132.21
53	33.79	53.52	29.38	79.29	25.16	103.95	21.40	125.92
54	36.14	47.66	31.49	72.62	26.93	97.09	23.02	118.07
55	38.66	42.16	33.77	66.21	28.87	90.31	24.51	111.76
56	41.32	37.10	36.22	60.03	30.98	83.60	26.23	104.96
57	44.21	32.23	38.90	54.01	33.29	77.02	28.09	98.34
58	47.28	27.75	41.77	48.30	35.80	70.57	30.18	91.53
59	50.54	23.75	44.86	42.96	38.55	64.31	32.46	84.91
60	54.05	19.94	48.20	37.83	41.51	58.28	34.97	78.28
61	57.72	16.70	51.78	33.06	44.74	52.46	37.68	71.91
62	61.65	13.72	55.61	28.66	48.22	46.94	40.64	65.68
63	65.81	11.13	59.71	24.62	51.98	41.72	43.88	59.64
64	70.18	8.94	64.10	20.92	56.03	36.82	47.39	53.84
65	74.81	7.06	68.77	17.61	60.38	32.35	51.19	48.32

NOTE ON METHOD OF PREPARING POLICY DATA FOR APPLICATION OF GROUP VALUATION COEFFICIENTS

The actuarial cards, or other records of the policies in force, are arranged according to year of issue, and, in the case of a card system, according to age at issue. Moreover, it is usual to separate these records according to sub-plans, that is to say, non-participating, participating by deferred dividends or by annual dividends, and so forth. For the purpose of the group valuation such subdivisions are not needed. All that is required is the total amount by age at issue and attained age, or, what is the same thing, by age at issue and duration. To place the amounts on the valuation sheets directly from the records would entail a very large amount of repetition, and this is the chief matter to be obviated. I have prepared the rough working sheet with the ages at issue at the side and the attained ages at the top for the purpose of collecting all similar policies, that is all policies on the same plan of insurance, such as ordinary life, limited payment life, and endowment independently of the sub-plans. The work entailed is trifling, as no rearrangement of original records is necessary, and the number of sheets usually used will not be more than six or seven. To illustrate in one case, say that of twenty payment life policies. A sheet is headed "Life 20" and from the first set of cards on this plan, say annually participating twenty payment life policies, the amounts, using $1,000 as the unit, are placed in the correct column and row. The succeeding plans are treated in the same way. Coming then to deferred dividend policies, on the twenty payment life plan, the amounts are placed on the same sheet as before used, and so on throughout all plans. At the end it will be found that the original cards have been handled once only and in their original order, also that the number of rough sheets used is only one for each distinct plan. Twenty to twenty-five millions of business can be placed on these rough sheets in two or three days.

Coming now to the valuation sheets. Commencing with any one of the rough working sheets, for example the ordinary life policies, the youngest attained age is entered at top of a valuation sheet and the amounts according to age at issue entered in the first column, following down the column of the working sheet. The same process is continued for each attained age. The next working sheet is then handled in precisely the same manner, the same

valuation sheets being used, new ones being inserted as new attained ages arise. In this manner the working sheets are used only once and as they come to hand.

When the figures from the last working sheet have been thus transferred, the whole record is on the valuation sheets, and there will be as many of these as there are different attained ages, and no more. This completes the record by plan and attained age.

From the tables of "Premiums and Premium Differences" the columns so headed are filled in, the sheets following both plan and order of the valuation sheets and so needing but one reference. In completion the totals are made and recorded at foot.

Finally each total is multiplied by its appropriate factor from Tabulation No. 1, the products being placed on the sheets to the extreme right-hand side under the caption "Products," and in accordance with their reference number, Nos. 1 and 3 are added, and No. 2 subtracted, the difference giving the reserve. In each operation from the initial one of treating the original records the method is direct, simple, and necessitates but once handling, and that directly, all cards and sheets.

NOTE ON INTRODUCTION OF D_{40}

Since preparing the tables for this paper it has been pointed out to me by Dr. D. F. Campbell that the introduction of this arbitrary constant is unnecessary. Its use is merely to render the arithmetical values of the factors N_{x+1} and $\frac{1}{D_{x+1}}$ convenient. This end will also be served by making use of the multiplier $\frac{100{,}000}{100{,}000}$, multiplying $\frac{1}{D_{x+1}}$ by 100,000 and dividing N_{x+1} by the same number. I am preparing tables to conform to the Illinois standard modified $3\frac{1}{2}$ per cent values and shall use Dr. Campbell's suggestion in the work.

QUESTIONS FOR SECTION "B" OF THE ASSOCIATE EXAMINATION

1. Find the probability expressed by ${}_{n-1}|\underset{x\ y}{q_1}$ and give the interpretation of it.

2. Give the formula for the annual premium of an ordinary life policy in terms of the annuity and interest.

3. Given ${}_nV_x=164.80$; $i=.035$; $q_{x+n}=.010829$; $P=17.19$; amount of insurance $1,000.00. Derive ${}_{n+1}V_x$.

4. On the basis of the following data, make the necessary entries on the books of record (accounting) of the company and show trial balance:

Capital stock paid in	$125,000
Surplus received on sale of stock	62,500
Premiums received on policies issued and paid for	26,880
First year commissions allowed agents	13,440
Fees for medical examinations	3,000
Fees for inspection of risks	1,000

5. Give the formula for the annual premium to secure an insurance beginning with 1 and increasing by 1 for t years and then all insurance ceasing.

6. Give the formula for the net annual premium for ordinary life insurance in terms of the D and N columns.

7. The sum of A is loaned at the interest rate of $i\%$ per annum convertible monthly. The loan, with interest, is to be repaid in 192 equal monthly installments. Find the amount of the monthly installment.

8. Give in commutation symbols a formula for the computation of the reserve at the end of the nth year on an m (<20) payment life policy issued at age x, on the basis of the Illinois standard.

9. Describe the various methods for loading the net premium in order to obtain the published rates.

10. Given the net premiums at each age for temporary increasing insurance (see Question No. 5), (*a*) It is required to compute from an ordinary life office rate the corresponding premium with return of all premiums paid in case of death within t years; deduce a formula to be used for this purpose. (*b*) Would you regard it as necessary

to load the extra premium required for this purpose. If so, how would you do it? Give details.

11. What investigation of a published commutation table would you make before using the N_x column?

12. What is the standard of valuation in the states of Ohio, New York, Illinois, and New Jersey?

13. Explain the terms Amortization and Accumulation of the value of Bonds.

14. Express the present value of a payment of 1 due in n years, provided x be then living and y shall have died t years previously; $n > t$, and rate of interest, i.

IN MEMORIAM

WHEREAS, BENJAMIN A. WELSTEAD, a Charter Fellow of the American Institute of Actuaries, was called from this life and work February 12, 1910, Therefore,

Be it resolved, That the American Institute of Actuaries has recognized and desires to record its appreciation of his work and character, extends its deepest sympathy to his family, and deplores the loss to his profession by his untimely death;

Be it further resolved, That this resolution be spread upon the minutes of the Institute and be published in *The Record* "In Memoriam," and that copies be sent to his family and the Michigan Mutual Life Insurance Company.